The Environment, Trade, and Innovation with Heterogeneous Firms: A Numerical Analysis

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1 Introduction

Research and development (R&D) in renewable energy resources as alternatives to fossil fuels is one of approaches to tackle the climate change. How to induce the R&D towards those technologies which would significantly reduce the greenhouse gas (GHG) emissions becomes one of the most pressing policy challenges facing the world today. Most of related empirical analysis in the environmental economics literature are either built upon computational general equilibrium models (Nordhaus, 1994), or assume exogenous technological improvement (Golosov et al. 2009), hence ignore the response of endogenous technical innovation to the environmental and/or related trade policies. Additionally, their theoretical frameworks based upon representative firm models are absence of the feature of firm's heterogeneity in terms of productivity, and do not characterize the stylized fact of firm's entry-exit decisions. However, these two features play substantial roles in explaining the firm-level economic activities from both theoretical and empirical international trade model (Melitz, 2003; Bernard, Jensen, Redding, and Schott, 2007; Bernard, Redding, and Schott, 2007; Atkeson and Burstein, 2010; etc).

A satisfactory framework for evaluating different environmental proposals and related trade policies must include at its centerpiece the endogenous response of technical improvement to proposed policies in the presence of heterogeneous firms. We adapt and extend the basic framework of heterogeneous firm and endogenous innovation activities developed in Atkeson and Burstein (2010) into a two-sector model with environmental performance. Two sectors are labeled as dirty and clean according to its sector-specific pollution intensity. Each produces a non-tradeable final good by combing a continuum of either domestically produced or exported intermediate inputs in that sector. Within each sector, intermediate firms with firm-specific productivity bears four types of costs, i.e. fixed entry costs, fixed operation costs, exporting costs including fixed and variable parts, and innovation costs. Upon observing these costs and productivity draws, they make decisions of entry, exit, production, export, and innovation. The innovation decision is modeled as an investment in improving the firm's productivity draw by choosing probabilities of success. These fixed cost structures together with the heterogeneous productivity determine that a continuum of heterogeneous intermediate firms is partitioned by the market status.

We perform simple numerical simulations concerning the implication of a stringent environmental policy and a proposal of trade cost differences on dirty and clean inputs. Our objective is to highlight the effects of these proposals on the technological innovation, trade pattern, and firm dynamics. We find that a symmetric reduction in emission permit cap raises the mass of entering firms in the clean sector, but lower the mass of entering firms in the dirty sector. There exists resource reallocation between sectors, the aggregate productivity across all domestic and exporting firms within the dirty sector drops, but that index within the clean sector rises. The proposal of trade variable costs differences on dirty and clean inputs has substantial impact on the technological innovation across sector. A relatively lower trade variable cost on the clean inputs contributes to investing in R&D activities for all clean firms, especially for those productive ones; On the contrary, a relatively higher trade variable cost on the dirty inputs discourages the dirty firms to engage in the technological innovation.

Our paper relates to the growing literature on growth, trade, and the environment. The pioneering study by Nordhaus (1994) introduces a dynamic integrated model of climate change and the economy, called the DICE model. Another branch of the literature focusing on the normative analysis includes a recent work by Golosov et al. (2009) and Acemoglu et al. (2011). The former studies the optimal policies in a model with exogenous technology and exhaustible resources, and it shows that the optimal resource tax should decrease over time. The latter one is built upon a growth model with endogenous and directed technical change. The innovation is directed by a group of scientist instead of firms themselves, hence it has the "state dependence" feature in the sense that advances in one sector make future advances in that sector more profitable or more effective. Their optimal policies involve a combination of a carbon tax, a subsidy to clean innovation and a subsidy for the use of all machines.

To my knowledge, there is a limited work applying the heterogeneous firms framework into the environmental economics. Li and Shi (2010) adapts and extends the closed economy model setup in the Melitz framework into environmental economics. They look into the efficiency assessment of alternative environmental policies between standard and tax, and find that productivity heterogeneity plays an important role in assessing the policies. Cui (2011) incorporates clean technology adoption and environmental performance into the Melitz framework. His study focuses on the implication of trade liberalization and stringent environmental policy on the exogenous clean technology adoption, and firm dynamics.

The remainder of the paper is organized as follows. Section 2 introduces the extended model setup, followed up by a characterization of simulated symmetric steady-state equilibrium in next section. Section 4 provides numerical results on stringent environmental policy and trade cost differences. Section 5 concludes.

2 Model Setup

The basic model setup is developed in Atkeson and Burstein (2010), and we extend it into a two-sector model in the presence of environmental activities.

2.1 Preference

An infinite-horizon discrete-time economy is inhabited by a representative consumer performing roles of a worker and entrepreneur. The representative consumer with an infinite life has preference over the composite consumption of non-tradable final goods, denoted by C_t, and values the quality of the environment, denoted by S_t,

$$\sum_{t=0}^{\infty} \frac{1}{(1+\lambda)^t} \left[U(C_t) - D(S_t)\right] \tag{1}$$

where $\lambda \in (0,1)$ is the discount rate. The separability of the per-period utility function allows the consumption to be exempt from the pollution externality. The damage function $D(S_t)$ is increasing and convex in the quality of the environment, S_t. At time t, the consumption of non-tradable final goods is composed of one final good produced in the dirty sector, called dirty good, indexed by C_{dt}, another produced in the clean sector, called clean good, indexed by C_{ct},

$$C_t = \left(C_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + C_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where $\varepsilon \in (1,\infty)$ denotes the elasticity of substitution between these two final outputs.

2.2 Production

At time t, a non-tradable final good $j \in \{c, d\}$, index clean and dirty respectively, is produced by a continuum of tradable intermediate goods in sector j. Intermediate good firms in each country are monopolistically competitive. Intermediate goods are each produced by heterogeneous firms indexed by the firm-specific productivity φ, which represents the quality of the intermediate good as well. Production of intermediate goods requires labor used as both fixed and variable costs. Let $f_j > 0$ be the time-invariant fixed production cost of serving domestic market, measured in labor units thereafter sunk. In addition, the production of intermediate goods also generates pollution byproducts. Following the technique by Copeland and Taylor (1995), we treat the emission byproduct as another input used in the production. Hence, an intermediate good firm with the firm-specific productivity φ in sector j at time t, produces intermediate output $y_{jt}(\varphi)$ according to the constant returns to scale production technology,

$$y_{jt}(\varphi) = \varphi^{1/(\rho-1)}(l_{jt})^{1-\beta_j}(e_{jt})^{\beta_j} \tag{3}$$

where l_{jt} is the units of labor used as variable costs; e_{jt} is the amount of pollution byproducts generated by the intermediate good of type φ; β_j denotes the time-invariant sector-specific emission intensity in sector j. For expositional convenience, we rescale firm productivity using the exponent $1/(\rho-1)$ such that each firm's equilibrium revenues and variable profits are proportional to φ, where $\rho > 1$.

International trade is subject to both time-invariant fixed costs of $f_j^x > 0$ measured in labor thereafter sunk, and iceberg type of variable costs, denoted by $\tau_j > 1$. Let $x_{jt}(\varphi) \in \{0, 1\}$ be an indicator of the export decision of home intermediate firms with φ in sector j at time t (with $x_{jt}(\varphi) = 1$ if the firm exports and 0 otherwise). We use an asterisk to distinguish foreign variables from the home ones when necessary, the corresponding foreign equations could be defined analogously.

The output of a home country intermediate good firm can be used to produce the home final good, with the quantity of this domestic absorption denoted by $a_{jt}(\varphi)$. Alternatively, some portion of its output could be exported to produce the foreign final good. The quantity of the output of the home intermediate firm used in the foreign country is denoted as $a_{jt}^x(\varphi)$. Since export is subject to variable costs of $\tau_j > 1$, the home intermediate firm must export $\tau_j a_{jt}^x(\varphi)$ units of output in order to have $a_{jt}^x(\varphi)$ units arrive in the foreign country for uses in the production of the foreign final good

j. Then, feasibility condition requires that,

$$a_{jt}(\varphi) + x_{jt}(\varphi)\tau_j a^x_{jt}(\varphi) = y_{jt}(\varphi) \tag{4}$$

Thus, a non-tradable final good in sector j is produced by assembling a continuum of home and foreign intermediate goods with a form of,

$$Y_{jt} = \left[\int a_{jt}(\varphi)^{1-1/\rho} dM_{jt} + \int x^*_{jt}(\varphi) a^{x*}_{jt}(\varphi)^{1-1/\rho} dM^*_{jt}\right]^{\rho/(\rho-1)} \tag{5}$$

where (M_{jt}, M^*_{jt}) denote the measures of the home and foreign intermediate firms in sector j at time t, respectively; $a^{x*}_{jt}(\varphi)$ denotes the units of the foreign intermediate goods which are exported and used for producing the home final good in sector; $x^*_{jt}(\varphi)$ is the export decision of the foreign intermediate firms. Intermediate goods are substitute with a constant elasticity of $\rho > 1$. Note that the first integration represents the home intermediate goods used in the domestic market, the second one expresses the foreign intermediate goods used in the export market. The production function form of non-tradeable final goods also captures the importance of both the quality and quantity of intermediate goods utilized in the production.

The non-tradeable final goods in both home and foreign countries are produced by competitive firms which choose output Y_{jt} and inputs $a_{jt}(\varphi)$ and $a^{x*}_{jt}(\varphi)$ subject to (5), to maximize profits given prices of the final good and intermediate goods P_{jt}, p_{jt}, p^{x*}_{jt}; export decisions $x_{jt}(\varphi)$, $x^*_{jt}(\varphi)$; and measures of operating intermediate firms M_{jt}, M^*_{jt},

$$\max_{a_{jt}, a^{x*}_{jt}} P_{jt}Y_{jt} - \int p_{jt} a_{jt}(\varphi) dM_{jt} - \int p^{x*}_{jt} x^*_{jt}(\varphi) a^{x*}_{jt}(\varphi) dM^*_{jt} \tag{6}$$

where the equilibrium price of a final good in sector j must satisfy

$$P_{jt} = \left[\int p_{jt}(\varphi)^{1-\rho} dM_{jt} + \int x^*_{jt}(\varphi) p^{x*}_{jt}(\varphi)^{1-\rho} dM^*_{jt}\right]^{1/(1-\rho)} \tag{7}$$

and the iso-elastic inverse demand curves in the domestic and export markets are:

$$a_{jt}(\varphi) = Y_{jt}\left(\frac{p_{jt}(\varphi)}{P_{jt}}\right)^{-\rho}; a^x_{jt}(\varphi) = Y^*_{jt}\left(\frac{p^x_{jt}(\varphi)}{P^*_{jt}}\right)^{-\rho} \tag{8}$$

A home intermediate firm with a firm-specific productivity draw φ faces a static profit maximization problem of choosing labor inputs $l_{jt}(\varphi)$, emission inputs $e_{jt}(\varphi)$, prices $p_{jt}(\varphi), p^x_{jt}(\varphi)$, quantities $a_{jt}(\varphi), a^x_{jt}(\varphi)$, and export decision $x_{jt}(\varphi)$, in order to maximize current period profits given the wage rate w_t, emission price p_{et}, prices and outputs of the final good j in both countries P_{jt}, P^*_{jt}, Y_{jt} and Y^*_{jt}. This static profit maximization problem is written as

$$\pi_{jt}(\varphi) = \max_{y_{jt},a_{jt},a^x_{jt},l_{jt},e_{jt},x_{jt}} p_{jt}a_{jt} + p^x_{jt}x_{jt}(\varphi)a^x_{jt}(\varphi) - w_t l_{jt}(\varphi) - p_{et}e_{jt}(\varphi) - w_t\left[f_j + x_{jt}(\varphi)f^x_j\right] \quad (9)$$

subject to (3), (4), (8). The optimal pricing rule is a constant mark-up over the marginal cost,

$$p_{jt}(\varphi) = \frac{c_{jt}(p_{et}, w_t)}{\sigma\varphi^{1/(\rho-1)}}; p^x_{jt}(\varphi) = \frac{\tau_j c_{jt}(p_{et}, w_t)}{\sigma\varphi^{1/(\rho-1)}} \quad (10)$$

where $\sigma \equiv 1 - 1/\rho \in (0,1)$, given $\rho > 1$; $c_{jt}(p_{et}, w_t)$ denotes the marginal cost of the home intermediate firm in sector j at time t. Due to the homogeneity of the production function (3), the marginal cost is given by:

$$c_{jt}(p_{et}, w_t) \equiv \mathbf{B}_j(w_t)^{1-\beta_j}(p_{et})^{\beta_j} \quad (11)$$

where $\mathbf{B}_j \equiv \beta_j^{-\beta_j}(1-\beta_j)^{\beta_j-1}$. The input demand functions across markets are derived using the Shepards' Lemma,

$$\begin{aligned} l_{jt}(\varphi) &= \frac{(1-\beta_j)\sigma}{w_t} r_{jt}(\varphi); \quad & l^x_{jt}(\varphi) &= \frac{(1-\beta_j)\sigma}{w_t} r^x_{jt}(\varphi) \\ e_{jt}(\varphi) &= \frac{\beta_j\sigma}{p_{et}} r_{jt}(\varphi); & e^x_{jt}(\varphi) &= \frac{\beta_j\sigma}{p_{et}} r^x_{jt}(\varphi) \end{aligned} \quad (12)$$

where $l_{jt}(\varphi)$ & $l^x_{jt}(\varphi)$ denote the variable labor input demand in the domestic and export market, respectively; $e_{jt}(\varphi)$ & $e^x_{jt}(\varphi)$ are the emission permit input demand in the domestic and export market, respectively.

Revenues earned from the domestic and export market, denoted by $r_{jt}(\varphi)$ and $r^x_{jt}(\varphi)$, respectively, are proportional to φ,

$$r_{jt}(\varphi) = Y_{jt}\left(P_{jt}\right)^{\rho}\left(\frac{c_{jt}(p_{et}, w_t)}{\sigma}\right)^{1-\rho}\varphi; r^x_{jt}(\varphi) = Y^*_{jt}\left(P^*_{jt}\right)^{\rho}\left(\frac{\tau_j c_{jt}(p_{et}, w_t)}{\sigma}\right)^{1-\rho}\varphi \quad (13)$$

We apportion the entire fixed production cost to the domestic market, the fixed exporting cost to the export market. Hence, total profits of a home intermediate firm in period t includes profits earned in the domestic market, denoted by $\pi_{jt}(\varphi)$, and profits earned in the export market, denoted by $\pi^x_{jt}(\varphi)$,

$$\pi_{jt}(\varphi) = \frac{r_{jt}(\varphi)}{\rho} - w_t f_j; \pi^x_{jt}(\varphi) = \frac{r^x_{jt}(\varphi)}{\rho} - w_t f^x_j \tag{14}$$

Thus, the equilibrium profits of the intermediate firm with productivity φ, denoted by $\Pi_{jt}(\varphi)$, can be written as,

$$\Pi_{jt}(\varphi) = \pi_{jt}(\varphi) + \max\left\{\pi^x_{jt}(\varphi), 0\right\} \tag{15}$$

The timing of the event is described as follows. At the beginning of each period t, in sector $j \in \{c, d\}$, each intermediate good firm pays a time-invariant fixed entrance fee of $f^e_j > 0$ as an initial investment to draw its firm-specific productivity φ from a common distribution function $g(\varphi)$ with a positive support. $g(\varphi)$ has a continuous cumulative distribution function of $G(\varphi)$. Upon observing the draw, the intermediate firm decides to operate a plant in that sector. If the firm does decide to operate, it bears a fixed production cost of $f_j > 0$ to establish a plant and serve the domestic market. Export requires an additional fixed cost of $f^x_j > 0$ and the standard iceberg form of variable cost $\tau_j > 1$. If the firm does produce, it also faces an exogenous probability $\delta \in (0, 1)$ of an idiosyncratic bad shock which forces it to exit. In the end of each period, the surviving firm with a productivity draw φ could invest $c(q)\varphi$ units of labor in R&D to improve its productivity. The R&D would succeed and raise the productivity by $\Delta\varphi$ with a probability q, it fails and hence suffer a productivity loss by the same amount otherwise. The firm's choice of q is referred as the process innovation in Atkeson and Burstein (2010), and the R&D expenditure of $c(q)$ is increasing and convex in q. A detailed function form will be specified in the later simulation section.

Let $V_{jt}(\varphi)$ be the value of an intermediate firm with productivity φ in sector j at time t after the realization of its productivity draw. Next period, the firm of type φ would survive with a probability of $1 - \delta$. Then, its productivity would be upgraded to $\varphi + \Delta\varphi$ with a probability of q, and be downgraded to $\varphi - \Delta\varphi$ otherwise. Given price sequences of $z = \{p_{et}, w_t\}$, the problem of an incumbent firm is defined recursively by a Bellman function:

$$V_{jt}(\varphi, z) = \max\Big\{\Pi_{jt}(\varphi) - c(q)\varphi + \frac{1-\delta}{1+\lambda}\max\left\{0, qV_{jt+1}(\varphi + \Delta\varphi, z) + (1-q)V_{jt+1}(\varphi - \Delta\varphi, z)\right\}\Big\} \tag{16}$$

In each period t and sector j, the decision of operating follows a cutoff rule that firms with productivity no less than a cutoff of $\hat{\varphi}_{jt}$ choose to operate and firms with productivity below that cutoff exit. Note that if the fixed production costs are assumed away, $f_j = 0$, then there is no endogenous entry and exit. Likewise for the export decision, given the static profit maximization problem, the export decisions are determined by the static condition that variable profits from exports must exceed fixed costs of exporting, that is,

$$x_{jt}(\varphi) = 1 \text{ iff } \pi^{x}_{jt}(\varphi) \geq 0 \tag{17}$$

In any period when new firms enter sector j after paying an initial entrance fee of f^e_j, free entry condition requires that

$$f^e_j = \frac{1}{1+\lambda}\int V_{jt+1}(\varphi, z)dG(\varphi) \tag{18}$$

where $1+\lambda$ also is the world interest rate.

Denote M^e_{jt} as the measure of potential new entrants of intermediate firms in sector $j \in \{c, d\}$ at time t. The measure of operating intermediate firms in the home country in period $t+1$ with state variable less than or equal to φ', denoted by $M_{jt+1}(\varphi')$, is equal to the sum of three inflows of firms: successful new entrants in period t; incumbents surviving from period t whose productivities are upgraded; and incumbents surviving from period t whose productivities are downgraded. This law of motion is written as follows:

For $\varphi' \geq \hat{\varphi}'_{jt+1}$ (19)

$$M_{jt+1}(\varphi') = M^e_{jt}[G(\varphi') - G(\hat{\varphi}'_{jt+1})] + (1-\delta)\int_0^{\varphi'-\Delta\varphi} q dM_{jt}(\varphi) + (1-\delta)\int_{\hat{\varphi}'_{jt+1}}^{\varphi'+\Delta\varphi}(1-q)dM_{jt}(\varphi)$$

For $\varphi' < \hat{\varphi}'_{jt+1}$, $M_{jt+1}(\varphi') = 0$

2.3 The Environment and Government

The quality of the environment in each country is being degraded by pollution emissions generated during the production of home intermediate goods. For simplicity, the global impact of pollution emissions is not accounted in the model. Hence S_t evolves according to the difference equation

$$S_{t+1} = (1+\theta)S_t - E_t \tag{20}$$

where θ is the rate of "environmental regeneration", E_t is the amount of pollution emitted from the dirty sector.

Government in each country implements a time sequence of pollution tax $\{p_{et}\}$, and would irrevocably precommit to it. The alternative emission permit cap-and-trade program would be also considered for policy comparison. Under this scenario, the government would set a time path of permit cap $\{\overline{E}_t\}$ instead. Intermediate firms must purchase the equivalent amounts of permits to emit pollution. Permits are not allowed to trade across country, neither does the inter-temporal trade. Revenues collected from auctioning emission permits would be transferred to the representative consumer in a lump-sum form. The feasibility condition for emission permits is,

$$E_t \equiv \sum_j \left\{ \int \left[e_{jt}(\varphi) + x_{jt}(\varphi) e^x_{jt}(\varphi) \right] dM_{jt} \right\} = \overline{E}_t \tag{21}$$

Labor inputs used in production as variable and fixed costs plus those sunk as initial entrance fees equal the labor endowment, hence labor market clearing condition is governed by,

$$L_t \equiv \sum_j \left\{ \int \left[l_{jt}(\varphi) + x_{jt}(\varphi) l^x_{jt}(\varphi) \right] dM_{jt} + \int \left[f_j + x^x_{jt}(\varphi) f^x_j + c(q)\varphi \right] dM_{jt} + f^e_j M^e_{jt} \right\} = \overline{L} \tag{22}$$

The aggregate revenue in the steady-state equilibrium equals the total payments to emission permits and labor inputs,

$$R_t = \sum_j \left\{ \int \left[r_{jt}(\varphi) + x_{jt}(\varphi) r^x_{jt}(\varphi) \right] dM_{jt} \right\} = p_{et}\overline{E}_t + w_t \overline{L} \tag{23}$$

2.4 Equilibrium

An equilibrium is a collection of sequences of aggregate prices and wages $\{R_t, R_t^*, P_{jt}, P_{jt}^*, w_t, w_t^*\}$, prices of emission permits $\{p_{et}, p_{et}^*\}$, prices of intermediate good $\{p_{jt}, p_{jt}^x, p_{jt}^*, p_{jt}^{x*}\}$, a collection of sequences of aggregate quantities $\{C_t, C_{jt}, Y_{jt}, C_t^*, C_{jt}^*, Y_{jt}^*\}$ and quantities of the intermediate goods $\{a_{jt}, a_{jt}^x, l_{jt}, l_{jt}^x, e_{jt}, e_{jt}^x, a_{jt}^*, a_{jt}^{x*}, l_{jt}^*, l_{jt}^{x*}, e_{jt}^*, e_{jt}^{x*}\}$, and a collection of sequences of firm value functions and profit, productivity cutoffs, and innovation decisions $\{V_{jt}, \Pi_{jt}, \hat{\varphi}_{jt}, \hat{\varphi}_{jt}^x, V_{jt}^*, \Pi_{jt}^*, \hat{\varphi}_{jt}^*, \hat{\varphi}_{jt}^{x*}, q_{jt}, q_{jt}^*\}$, and measures of operating and entering firms $\{M_{jt}, M_{jt}^e, M_{jt}^*, M_{jt}^{e*}\}$, and quality of environment $\{S_t\}$ such that, in each period and each country:

(i) representative household maximizes her utility subject to the budget constraint;

(ii) intermediate good firms maximize within-period profits;

(iii) final good firms maximize profits;

(iv) labor and emission permit input markets clear, respectively, (21), (22);

(v) mass of operating firms and evolution of environmental quality are given by the law of motion, respectively, (19), (20).

3 Simulation

In this section, we perform numerical simulations assuming the symmetric steady-state equilibrium in which all of the variables are constant, and countries are symmetric. The transition dynamics is omitted in the current stage. Our objective is to highlight the effects of environmental policy and related trade proposals on the technological innovation and firm dynamics in the steady-state equilibrium. In general, parameters are chosen to make our exercises as similar to existing quantitative analysis as possible, i.e. Acemoglu et al. (2011), Atkeson and Burstein (2010), Bernard, Redding and Schott (2007), etc. Their numerical calibrations reproduce a number of salient features of US data on firm dynamics, international trade, production of nonfossil and fossil fuel sectors, and atmospheric concentration of carbon dioxide (CO_2).

3.1 Symmetric Steady-State Equilibrium

In our simulation analysis with a symmetric steady-state equilibrium, the export variable profit is related with the domestic variable profit, $\pi^{vx}_{jt} = \pi^{v}_{jt}\tau_j^{1-\rho}$. Now assume that the firm's exit, export decisions and research decisions are given and the associated steady-state distributions per entering firms across sectors $\tilde{M}_{jt}(\varphi) \equiv M_{jt}(\varphi)/M^e_{jt}$ are given. The time subscript is omitted. To solve for the remaining aggregate variables, we first define several measures of aggregate productivity. Let $\tilde{\varphi}^d_j$ be an index of productivity aggregated across all operating and non-exporting home intermediate firms; and $\tilde{\varphi}^x_j$ be an index of productivity aggregated across all exporting home intermediate firms,

$$\tilde{\varphi}^d_{jt} \equiv \int [1 - x_{jt}(\varphi)]\varphi d\tilde{M}_{jt}; \tilde{\varphi}^x_{jt} \equiv \int x_{jt}(\varphi)\varphi d\tilde{M}_{jt} \tag{24}$$

Both indexes are scaled by the mass of entering firms. Hence, our ideal measure of aggregate productivity in home country, $\tilde{\varphi}_{jt}$, is given by,

$$\tilde{\varphi}_{jt} = M^e_{jt}\left[\tilde{\varphi}^d_{jt} + (1 + \tau_j^{1-\rho})\tilde{\varphi}^x_{jt}\right] \tag{25}$$

By symmetry, $\tilde{\varphi}_{jt}$ also represents the aggregate productivity of all operating firms (domestic and foreign) competing in home country (where the productivity of exporters is adjusted by the trade cost τ). Put it differently, $(\tilde{\varphi}_{dt}, \tilde{\varphi}_{ct})$ correspond to the aggregate "dirty sector-specific technology" index and "clean sector-specific technology" index, respectively.

From firm's static profit maximization problem, we have that the aggregate emission permit inputs used in production of home intermediate firms in sector j in a symmetric steady-state is given by,

$$E_j = \frac{\beta_j \sigma}{w} Y_j P_j^{\rho} \left(\frac{c_j}{\sigma}\right)^{1-\rho} \tilde{\varphi}_j \tag{26}$$

Similarly, the aggregate variable labor inputs used in production of home intermediate firms in sector j is given by,

$$L_j = \frac{(1-\beta_j)\sigma}{w} Y_j P_j^{\rho} \left(\frac{c_j}{\sigma}\right)^{1-\rho} \tilde{\varphi}_j \tag{27}$$

The average labor inputs per entering firms used as fixed costs and R&D investments of home intermediate firms in sector j, which we denote by l_j^f, are written as,

$$l_j^f = \int \left[f_j + x_j(\varphi) f_j^x + c(q)\varphi \right] d\tilde{M}_j + f_j^e \tag{28}$$

Thus, the aggregate labor inputs used as fixed costs, indexed by L_j^f, is given by $L_j^f = M_j^e l_j^f$.

For sector $j \in \{c, d\}$, given π_j^v, $\tilde{\varphi}_j$, and l_j^f, the symmetric steady-state values of p_e, w, R, Y_j, C_j, P_j, and M_j^e solve the following system of equations,

$$\begin{aligned}
P_j &= \left(\frac{c_j}{\sigma}\right) \tilde{\varphi}_j^{1/(1-\rho)}, \text{ for j } \in \{c, d\} \\
Y_j &= \pi_j^v \rho \left(\frac{\sigma}{c_j}\right)^{1-\rho} P_j^{-\rho}, \text{ for j } \in \{c, d\} \\
C_j &= R P_j^{-\varepsilon} / (P_c^{1-\varepsilon} + P_d^{1-\varepsilon}), \text{ for j } \in \{c, d\} \\
Y_j &= C_j, \text{ for j } \in \{c, d\} \\
E &= \sum_j \left\{ \frac{\beta_j c_j}{p_e} Y_j \tilde{\varphi}_j^{1/(1-\rho)} \right\} = \overline{E} \\
L &= \sum_j \left\{ \frac{(1-\beta_j) c_j}{w} Y_j \tilde{\varphi}_j^{1/(1-\rho)} + M_j^e l_j^f \right\} = \overline{L} \\
R &= p_e \overline{E} + w \overline{L}
\end{aligned}$$

3.2 Algorithm

The recursive algorithm using the Matlab code to look for the symmetric steady-state equilibrium is similar to the algorithm used in Atkeson and Burstein (2010), and is described as follows.

First step, given permit price p_e (wage rate is normalized to one), variable domestic profit π_j^v, and mass of entering firms across sector M_j^e as initial guesses, we first pass the initial guesses of π_j^v into firm's dynamic program problem for each sector using the value function iteration method. We find out the level of π_j^v that is consistent with free entry condition, which in turn gives exit and export decisions.

Then associated with π_j^v are the firm's policy functions which are used to find the stationary distribution of firms according to the law of motion. With these policy functions and stationary distribution, the remaining aggregate variables could be written as functions of policy's functions, stationary distribution, initial value of permit price, and mass of entering firms.

Finally, we use "fsolve" in Matlab to find out the equilibrium values of p_e, w, and M_j^e such that two final goods market clearing, emission permit and labor market clearing conditions hold.

3.3 Parameters

Table 1 summarizes all of baseline parameters. In general, most parameters are drawn from Atkeson and Burstein (2010), Bernard, Redding and Schott (2007) and Acemoglu et al.(2011). Both dirty and clean sectors share the same values of fixed production costs, entry costs and exporting costs, all of which come from Atkeson and Burstein (2010) and Bernard, Redding and Schott (2010), the latter focuses on a two-sector general equilibrium model with comparative advantage. However, sectors differ only in pollution intensities. We take a value of $\beta_d = 0.6$ for pollution intensity in the dirty sector, of $\beta_c = 0$ in the clean sector assuming the absolutely clean sector for simplicity. The exogenous exit rate of 0.55% is drawn from Atkeson and Burstein (2010), they find that this value is consistent with that rate for large firms in the U.S. data. The annual interest rate (the annual discount rate as well) assumes a value of 5% from Atkeson and Burstein (2010). The value of intermediate goods elasticity of substitution assumes $\rho = 5$. Acemoglu et al. (2011) uses a value of $\varepsilon = 3$ for the output production elasticity of substitution. Their model assumes that the production process employs dirty and clean goods as inputs. While, we consider the same value of $\varepsilon = 3$ for the elasticity of substitution between aggregate clean and dirty goods from the consumption aspect.

The productivity distribution G is parameterized such that all firms enter with a common productivity of $\varphi = 0$, a discrete productivity shock assumes $\Delta\varphi = 0.25$. As in Atkeson and Burstein (2010), the process innovation cost function adopts a form of $c(q) = e^{bq}$, where b governs the curvature of this function. This curvature parameter also represents the elasticity of innovation, the higher value of b, the more inelastic the process innovation decision is. In the baseline, we consider the same low value of $b = 10$ for both dirty and clean sectors, so that the reallocation of process innovation is quite large if a trade cost changes. The last key parameter is the discount rate, which adopts the Stern discount rate of $\lambda = 0.014$ per annum.

4 Numerical Results

In this section, we conduct two numerical experiments concerning the implications of a stringent environmental policy and a scenario of trade cost differences. These computational exercises aim to highlight their impacts on the process innovation, trade pattern, and productivity dynamics within and between sectors. Calibration exercise along with an asymmetric steady-state equilibrium is beyond the scope of the current version of this paper.

4.1 Stringent Environmental Policy

We consider a reduction in emission caps by 20% below the baseline value of $\overline{E} = 80$. The transition dynamics is not accounted in the paper, we only show changes of the symmetric steady-state equilibrium in response to a reduction in cap. Figure 1 depicts the implications of the stringent environmental policy on prices, mass of entering firms, aggregate productivity, and aggregate production/consumption. The horizontal axis from right to left describes a reduction in emission cap. The solid line is for the clean sector, the red dash-dot line refers to the dirty sector. Numbers in the vertical axis are not meaningful in the absolute value unless the model is well calibrated to reflect the U.S. data.

The model in the aggregate level is in the context of the Heckscher-Ohlin trade model. As predicted in the Rybczynski theorem, a reduction in emission cap would decrease the aggregate production of the emission-intensive sector (dirty sector); and increase the aggregate production of the labor-intensive sector (clean sector).

The heterogeneous firm's framework could shed lights on the policy implications in much broader dimensions than the traditional trade model with homogeneous firms. As usual, permit price is decreasing in emission cap level. A tougher environmental policy raises the permit price, which in turn affects all operating firms with different magnitudes. As a consequence, the relatively less productive dirty plants are driven out, since they are unable to earn enough revenues to cover production costs. The surviving dirty plants charge higher prices for tradeable intermediate goods, then put upward pressure on the aggregate price for dirty goods. The aggregate production in dirty sector falls as both production costs keep rising and less productive plants shut down. However, plants in the clean sector are exempt from the upward pressure on emission permits.

Another advantage of using the heterogeneous firm model is to characterize the impacts on the mass of entering firms and aggregate productivity across all operating domestic and foreign intermediate firms, as shown in the bottom panel of Figure 1. According to the recursive algorithm described in the pervious section, changes other than fixed production costs and trade costs have no effects on firm's innovation decision, henceforth the steady-state distribution normalized by entering firms.[1] A reduction in emission cap in this particular framework would only affect firm's productivity dynamics through its influence on the mass of entering firms. Thus, changes of the mass of entering firms also reflect changes of the aggregate productivity indices. The fiercer competition in the emission permit market requires potential new dirty plants to draw higher productivity, illustrated by a falling mass of entering firms in the dirty sector. As expected, the aggregate productivity of all operating firms in the dirty sector falls as the emission permit cap declines. Likewise for the aggregate productivity indices for exporters and non-exporters, which are not shown in Figure 1.

4.2 Trade Cost Difference

We are interested at a particular scenario in which trade variable costs differ across sectors. Specifically, intermediate inputs in the dirty sector are subject to relatively higher trade variable costs than those in the clean sector. It illustrates a potential international trade agreement devoted to the clean environment. For example, an additional carbon fee for all carbon related intermediate inputs which are shipped across countries. Under this numerical scenario, around 20% more trade variable cost are charged for all exported dirty intermediate inputs. Figure 2 illustrates the scatter plots of the value function and process innovation against the productivity grid. The blue cross marker indicates the clean sector, the red point marker refers to the dirty sector.

With a 20% difference of trade variable costs between sectors, as shown in the upper panel of Figure 2, intermediate firms in the clean sector have higher values of operation as compared to those in the dirty sector, and the gap of operation value between sector rises as productivity increases. Such gap arises only from trade variable cost differences since pollution intensity varying

[1] The Bellman equation (16) only depends on initial guess of variable profit, discount rate, and cost structures including production, exporting and innovation process. As long as all these costs are measured in units of labor, price system like permit price, wage rate or aggregate price would not come into play in solving this dynamic programming problem by the value function iteration method. Thus, a changing permit price due to a reduction in emission permit cap would have no impacts on the process innovation and value function due to this special modeling assumption.

across sector does not come into effect in solving firm's dynamic programming problems. A higher trade variable cost sets higher trade barriers on intermediate inputs produced in the dirty sector, and make those dirty plant much harder to survive in the exporting market. Only those relatively productive dirty plants could still choose to export, the rest of them have to serve only the domestic market. This resource reallocation from the exporting market to the domestic one lowers the value of exporting, hence the value of operation, which in turn discourages the process innovation among the dirty plants. Standing in the contrast, a relatively lower trade variable cost on clean inputs contributes to the exporting market for the clean plants. As a consequence, it increases the incentive of engaging in the process innovation for the clean plants, especially for those exporting ones. Trade cost differences encourage the process innovation in the clean sector, but discourage that in the dirty one, as captured in the lower panel of Figure 2.

Table 2 and 3 list simulation results about key variables in this system for two scenarios differing in trade variable costs. In the baseline scenario showed in Table 2, the fixed trade cost is assumed to be equal cross two sectors at a value of 0.30, while the trade cost adjusted term ($\tau^{1-\rho}$) in the dirty sector is changed into 0.15, which implies an increase in the trade variable cost in the dirty sector as in Table 3.[2]

In the baseline scenario, there are approximately 78% firms in both sectors involving in the exporting activities and the average R&D rate in both sectors is around 43%. The equalities between two sectors can be attributed to the specific model setup and algorithm we used to solve the systematic steady-state firm distribution.[3] Since the dirty firms bear some carbon fees due to the emission cap, the price for final dirty output is a little bit higher than the price of clean final output. Due to this exact reason, the mass of entering firms, the labor utilized in the clear sector (both for production and firm built-up), the total production of clear final output and the aggregate productivity (total productivity, domestic productivity and exporting productivity) in the clean sector are all larger than the counterparts in the dirty sector.

When the trade cost in the dirty sector increases, the portion of exporting firms in the dirty sector drops dramatically from 78% to barely 15%, an almost 80% drop. At the same time, the average

[2]The economic system is not calibrated to any real economy and the simulated policy change is not to mimic any real policy proposals, thus the results should be thought as a qualitative analysis rather than a quantitative analysis. In the future work, we would like to use the U.S. data to calibrate the model and provide a quantitative analysis on the related policy.

[3]The reason is similar to the one discussed in aforementioned session.

R&D rate decreases to 38% (an 11 percent drop from the baseline rate). These decreases come from the fact that the increase in trade cost discourages the exporting activities at all productivity level and lowers down the value of firms at all productivity levels, which in turn leads to less exporting firms and lower R&D activities. The prices for final goods, aggregate production, aggregation production labor and aggregate labor in both sectors do not change significantly compared with the baseline scenario. However, the mass of entering firms in the dirty sector increases up to 3.38 from 1.48. This contrast implies that the relatively high productive exporting firms are replaced by the relatively low productive domestic firms, since high productive firms need more labor than low productive ones, a big mass of low productive firms is needed to support the same amount of production labor in the dirty sector. This change in landscape of firms in the dirty sector is also reflected by the aggregate productivity. Although the aggregate productivity changes slightly in both sectors, there are big differences in the allocation between aggregate domestic productivity and aggregate exporting productivity. Compared with the baseline distribution, the aggregate domestic productivity in the dirty sector increases from 5.199 to 24.136, while at the same time the aggregate exporting productivity drops from 25.640 to 4.275. The increase of trade cost in the dirty sector leads to significant changes in the distribution of firms between the domestic and exporting firms, and modest changes in the production of final good in both sectors.

5 Conclusion

This paper extends the basic framework in Atkeson and Burstein (2010) into a two-sector model with environmental constraints. Two sectors, clean and dirty, employ a continuum of tradeable intermediate inputs produced from either home intermediate firms or foreign ones to create the non-tradable final outputs, labeled as clean and dirty good, respectively. Sectors differ in the sector-specific pollution intensity. Intermediate firms must purchase the equivalent amount of emission permits from their home country's government to emit pollutants. Intermediate firms with the firm-specific productivity bears four different types of costs, i.e. fixed entry cost, fixed operation cost, exporting cost including both fixed costs and iceberg form of variable costs, and the process innovation costs.

We perform several numerical simulations to highlight the effects of a stringent environmental policy and trade cost differences on the process innovation, trade pattern, mass of entering firms, and aggregate productivity indices. As expected, a symmetric reduction in emission permit cap raises the mass of entering firms in the clean sector, but lower the mass of entering firms in the dirty sector. There exists resource reallocation between sectors, since dirty plants become much more difficult to survive under the high pressure of emission permit price than clean plants. Consequently, the aggregate productivity across all domestic and exporting firms within the dirty sector drops, but that index within the clean sector rises. Another numerical experiment concerns the impacts of trade variable cost differences across sector. A lower trade variable cost on the clean intermediate inputs contributes to R&D activities for all clean firms, especially for those productive ones; On the contrary, a higher trade variable cost on the dirty intermediate inputs discourages the dirty firms to engage in the process innovation.

The current version of this paper is absence of an accurate calibration along with an asymmetric steady-state equilibrium in which countries are different in terms of endowments, sector-specific technology, etc. In the future research agenda, we would perform several more numerical simulations regarding an asymmetric stringent environmental policy or trade liberalization. These policy changes occur in the way of either an unilateral reduction in emission cap or trade cost on clean good.

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Table 1: Parameters

Variables	values	sources
Fixed production costs	$f_c = f_d = 0.1$	AB(2010)[1], BRS(2007)[2]
Sunk entry costs	$f_c^e = f_d^e = 1$	AB(2010)
Fixed exporting costs	$f_c^x = f_d^x = 0.13$	AB(2010), BRS(2007)
Variable exporting costs	$\tau_c = \tau_d = 1.3$	AB(2010)
Industry factor intensities	$\beta_c = 0$, $\beta_d = 0.6$	assumed
Factor endowments	$\overline{L} = 100$, $\overline{E} = 80$	assumed
Elasticity of process innovation	$b_c = b_d = 10$	AB(2010)
Productivity jump	$\Delta\varphi = 0.25$	AB(2010)
Exit rates	$\delta = 0.0055$	AB(2010)
Input elasticity of substitution	$\rho = 5$	AB(2010)
Output elasticity of substitution	$\varepsilon = 3$	Acemoglu et al.(2011)
Discount rate	$\lambda = 0.014$	Stern(2006)

[1] Atkeson and Burstein, 2010

[2] Bernard, Redding, and Schoot, 2007

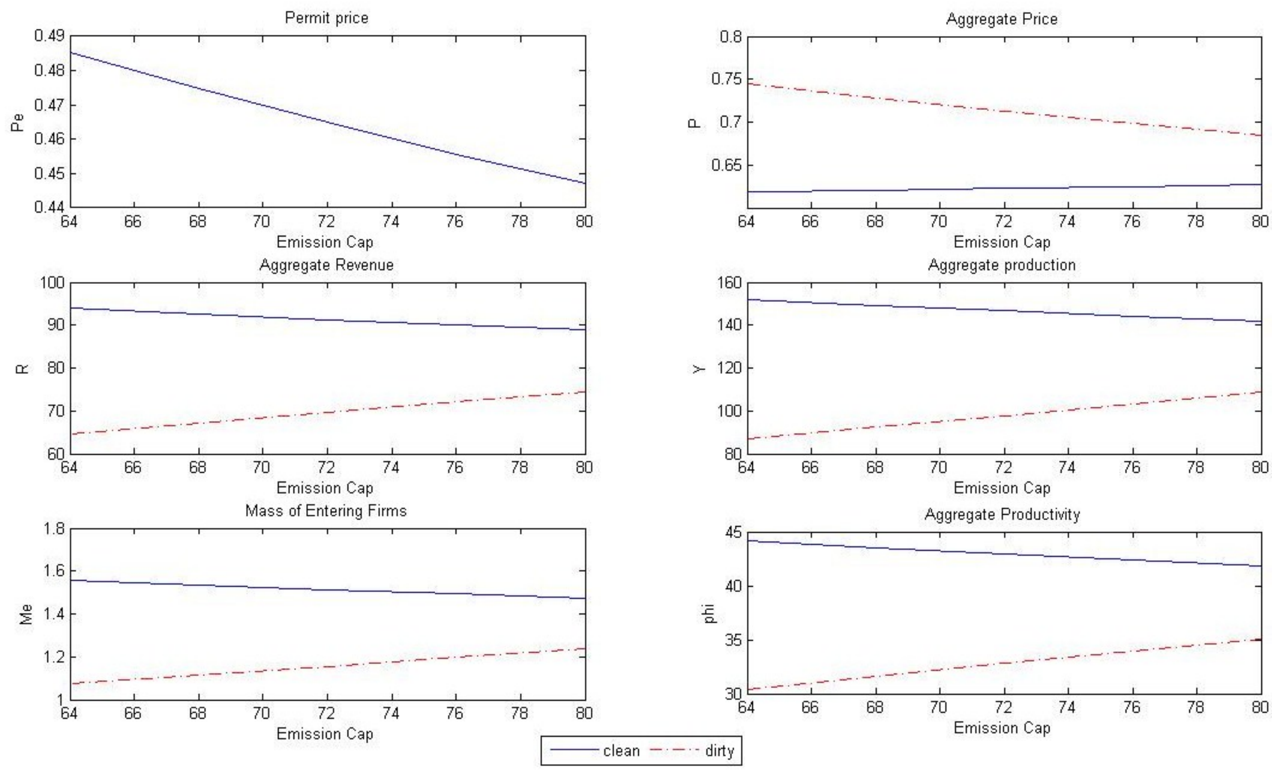

Figure 1: The Implication of a Tougher Environmental Policy

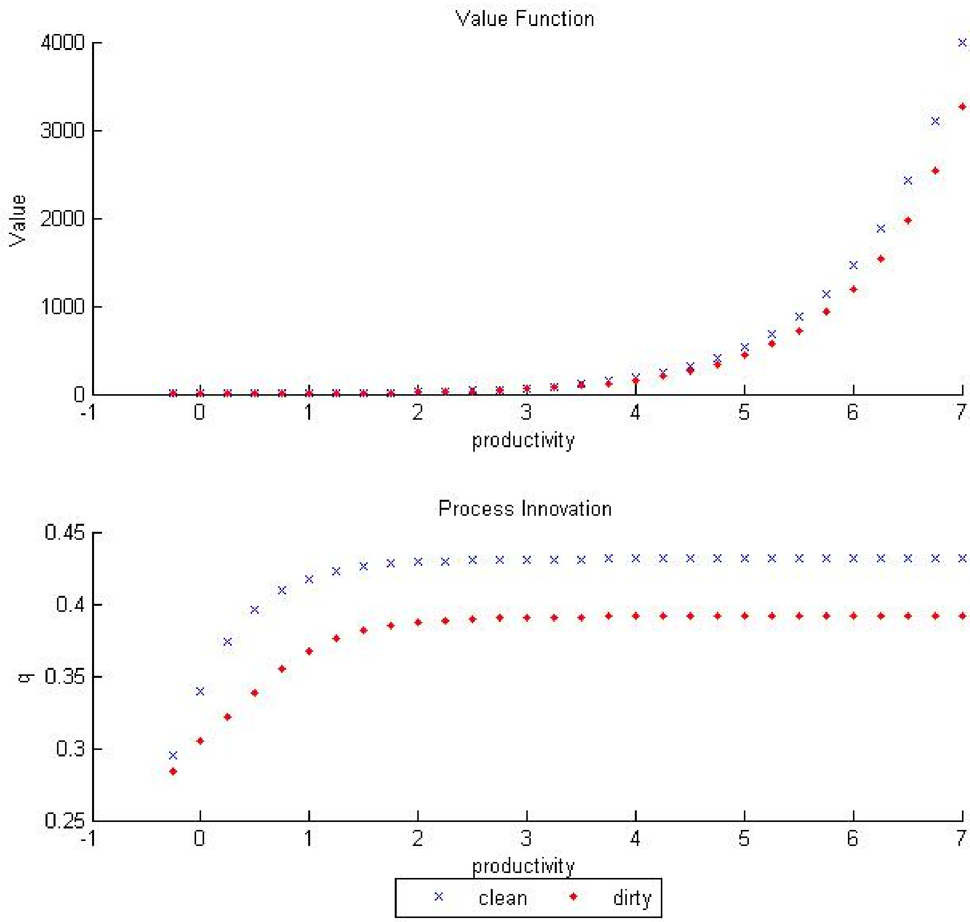

Figure 2: Effects of Trade Variable Cost Differences across Sector

Table 2: Selected Steady-State Equilibrium Variables in Baseline

	variable	clean	dirty
Trade variable cost	$\tau^{1-\rho}$	0.30	0.30
Export share		0.784	0.784
Average process innovation	q	0.429	0.429
Constant on variable profits	π^v	0.434	0.434
Aggregate price	P	0.582	0.635
Aggregate production	Y	137.154	105.386
Aggregate production labor	L_p	55.701	18.691
Aggregate Labor	L	69.627	30.374
Mass of entering firms	M^e	1.766	1.482
Aggregate productivity	$\tilde{\varphi}$	38.760	30.839
Aggregate domestic productivity	$\bar{\varphi}^d$,[1]	6.197	5.199
Aggregate exporting productivity	$\bar{\varphi}^x$,[2]	30.563	25.640

[1] $\bar{\varphi}^d \equiv M_j^e \tilde{\varphi}_j^d$

[2] $\bar{\varphi}^x \equiv M_j^e (1 + \tau_j^{1-\rho} \tilde{\varphi}_j^x)$

Table 3: Effects of Trade Variable Cost Differences across Sector

	variable	clean	dirty
Trade variable cost	$\tau^{1-\rho}$	0.30	0.15
Export share		0.831	0.150
Average process innovation	q	0.429	0.380
Constant on variable profits	π^v	0.425	0.443
Aggregate price	P	0.567	0.624
Aggregate production	Y	136.833	102.820
Aggregate production labor	L_p	55.952	18.498
Aggregate Labor	L	69.502	30.498
Mass of entering firms	M^e	1.718	3.378
Aggregate productivity	$\tilde{\varphi}$	35.768	28.410
Aggregate domestic productivity	$\bar{\varphi}^d$,[1]	6.030	24.136
Aggregate exporting productivity	$\bar{\varphi}^x$,[2]	29.739	4.275

[1] $\bar{\varphi}^d \equiv M_j^e \tilde{\varphi}_j^d$

[2] $\bar{\varphi}^x \equiv M_j^e (1 + \tau_j^{1-\rho} \tilde{\varphi}_j^x)$